Determinants of efficient growth boundaries with balanced budgets and stochastic rents

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Abstract

This article extends Ding et al. (1999) to investigate the design of growth boundary controls in an open city in a framework where a congestible public good is priced at average cost. Unlike Ding et al., we consider that urban rents evolve stochastically over time and that the regulator can slacken the boundary control when urban rents move upward. We provide three novel testable implications, namely, the regulator will set a more stringent boundary control policy if (i) developers expect urban rents to grow less rapidly; (ii) developers expect urban rents to be more volatile; and (iii) it costs less for the regulator to implement the control policy.

Keywords: balanced budgets, growth boundaries, real options, stochastic rents

JEL Classification: G13; H23; R52; R58

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I. Introduction

As a city boundary expands with population growth, supply of public goods may become a financial burden for the city government because the cost for supplying them may increase substantially with spatial expansion. At the same time, the city government may need to maintain balanced budgets such that it may opt to price public goods at average cost and simultaneously restrict the city boundary.[1]

While Ding, Knaap, and Hopkin (1999) have investigated the design of growth boundary controls in a monocentric open city framework where a congestible public good is priced at average cost, we depart from Ding et al. in the following respects. First, Ding et al. assume that urban rents grow deterministically over time, such that the date at which land is developed is also non-stochastic. In contrast, we assume that urban rents evolve stochastically and, thus, the development timing is also

[1] Urban sprawl can have many adverse effects on local communities as a result of car-dependent communication on the urban fringe, congested highway systems, and less green-space (Juergensmeyer and Roberts, 2003). Consequently, many local communities employ growth management controls to rectify these negative impacts. These control programs include: The *time* or *phased* growth control, as developed by Ramapo, New York; the *population control* strategy listed in the Boca Raton plan in Florida; and the urban growth boundary control employed in the Oregon and Minnesota states. Alternatively, some communities employ policies to raise the cost of development in order to curb growth. These policies include municipal exactions, regulatory delays in the approval process (Mayo and Sheppard, 2001), and impact fees. See the review paper by Brueckner (2007), which investigates the effects of three commonly employed interventions, including urban growth boundaries, FAR (floor area ratio) restrictions, and cost-increasing regulations.

stochastic. Second, Ding et al. assume that the growth control policy is time invariant, while we allow the regulator to have the option to slacken the control when urban rents move upward. Our assumption is more consistent with the existing land-use planning system, which requires local governments to periodically change city boundaries as population grows (see, e.g., Ding et al., 1999; Jaeger and Plantinga, 2007). Finally, we assume that the regulator incurs costs to implement the control policy, while Ding et al. abstract from them.

Similar to Ding et al., we find that the regulator will set a more stringent boundary control policy if either the development cost, the agricultural rent, the commuting cost, or the stock of infrastructure increases. Our paper contributes to the literature by pointing out that the regulator will tighten the boundary control if developers expect urban rents to grow less rapidly or become more volatile, or if the implementation cost decreases.

This article assumes that the regulator may need to use growth boundary controls to correct inefficiencies associated with the pricing of a public good. This contrasts with several articles that employ the amenity-creation model, which assumes that development exerts a negative externality on city residents. These articles include Cooley and LaCivita (1982), Brueckner (1990), Engle, Navarro and Carson (1992), Helsley and Strange (1995), Sakashita (1995), Brueckner and Lai (1996), Sasaki

(1998), Turnbull (2004a), and Jou and Lee (2008). This article also contrasts with several articles that employ the supply-restriction model, which argues that housing price increases with the decrease of housing supply created by growth controls. See, for example., Brueckner (1995) and Brueckner and Lai (1996).

The remaining sections are organized as follows. The next section presents the basic assumptions of our model. In Section III, we derive the choice of development timing and the city boundary in competitive equilibrium as well as the rule for a regulator to design the efficient boundary control. In Section IV, we employ plausible parameters to investigate how various forces that are related to the demand and supply conditions of the real estate market affect both the equilibrium and the efficient city boundary. The final section concludes and suggests directions for future research.

II. The Model

Our basic model extends the models of Capozza and Helsley (1990) and Ding et al. (1999). Consider a linear open city with all employment (the CBD) located at one end. The city has perfectly competitive factor and product markets. Residents located D miles from the CBD must allocate their income, $y(t)$, so as to pay for the numeraire good, land rents, R, and transportation costs θD, where θ (>0) represents the commuting cost per mile. At date $t=0$ an absentee landowner, who is also a risk-neutral developer, faces a discount rate denoted by ρ.[2] The landowner has a parcel of vacant land (normalized at one unit) at location D, which yields a rent equal to γ (≥ 0), such that it is never optimal for the landowner to abandon the vacant land. At any time $t \geq 0$, the landowner is able to develop the vacant land at the cost of F, which is fully irreversible.[3]

As in Ding et al. (1999), we assume that there exists a congestible public good, z, which must be provided to all residents of the urban area, but is rival in consumption. Examples of these kinds of goods include education, water and waste-water services, and police and fire protection (Ding et al., 1999). The public good is produced with a fixed stock of urban infrastructure, k, and the variable input, m, such that the public good production function can be specified as

$$z = G(k, m, n), \tag{1}$$

where n represents the urban population and $\partial G / \partial m > 0$ and $\partial G / \partial n < 0$, implying that the level of the public good rises with variable inputs and decreases

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[2] We can generalize our model into a risk aversion environment in the manner of Cox and Ross (1976).
[3] McFarlane (1999) argues that investment on land development will be fully irreversible if demolition costs are extremely high. Similarly, Riddiough (1997) suggests that irreversibility is a reasonable assumption for real estate, in which the physical asset is long-lived and switching costs to alternative uses are quite high. Turnbull (2005) argues that the irreversibility assumption may not be realistic, but provides an analytically tractable solution.

with population.

Assuming that input prices are unit, and that local governments adjust the variable input to maintain z fixed at a constant level, $\overline{z}$, a public good variable cost function may be specified as

$$m = n^2 k . \tag{2}$$

Equation (2), which indicates that the cost of maintaining a constant level of the public good rises with population, resembles that used by Brueckner (1997) and in the literature on clubs.

Following Ding et al., we assume that local governments set the price of a congestible public good at average cost rather than marginal cost so as to yield a balanced budget. Consequently, city residents may become better off when the city population is reduced through the application of a growth boundary control. The control has no effect on the utility level of consumers, however, because we assume that the city is too small. All consumers have identical utility functions, which have as arguments a numeraire good, c, land, ℓ, and the public good, z. If each resident consumes one unit of land, the price of the numeraire good is one, and the costs of the public good are paid by landowners, then using the budget constraint, we can construct a representative utility function as given by

$$v(c,\ell,z) = v(y(t)) - R - \theta D, \ell, z) = u(t) , \tag{3}$$

where under open city assumptions, resident utility must equal exogenous utility levels. From Equation (3), we can construct a bid rent function as given by

$$R(D, x(t)) = \eta + x(t) - \theta D, \tag{4}$$

where $y(t) = x(t) + \eta + c$, and the stochastic factor $x(t)$ follows the arithmetic Brownian motion

$$dx(t) = \alpha dt + \sigma d\Omega(t),\ \alpha > 0, \quad \sigma > 0, \tag{5}$$

where α is the constant drift rate, σ is the instantaneous volatility of $x(t)$, and $d\Omega(t)$ is an increment to a standard Wiener process.[4] The specification of Equation (4) indicates that the income of residents is the sum of two parts: the fixed part of $\eta + c$ and the stochastic part of $x(t)$.

Our assumptions differ from those of Ding et al. (1999) in the following respects. First, we assume that urban rents evolve stochastically as in Equation (5), while Ding et al. assume that these rents are deterministic. Second, we allow a regulator to set a time-contingent boundary control policy, while Ding et al. assume that the control is

[4] Our specification is the same as that assumed in Capozza and Helsley (1990), Capozza and Li (1994), and Capozza and Sick (1994). The arithmetic Brownian motion of urban rents is supported by the empirical data, but suffers the shortcoming that urban rents may be negative. See a thorough discussion in Capozza and Li (1994).

time invariant.[5] Finally, we assume that the regulator incurs costs to implement the control policy, while Ding et al. abstract from them. With these differences, we can derive several testable implications that are novel to the literature, as shown later.

Given that the public good is inefficient priced, local governments can thus implement two broad types of policies to correct this (Turnbull, 2004a): (i) Fees, or special exactions, imposed on newly developed property; and (ii) direct quantity restrictions in the form of urban growth boundaries, greenbelt preserves, land banks, or development moratoria. We will focus only on the design of growth boundary controls.

An individual landowner will choose a date at which to develop vacant land. The social planner, who anticipates the development timing chosen by the developer, can set an efficient level for the city boundary. We will sequentially investigate the developer's and the social planner's decision in the following section.

III. Choices of Development Timing

We assume that a regulator implements a growth boundary control policy at the current time $t = 0$. Therefore, this policy is anticipated by developers when they

[5] Ding et al. (1999) allow the regulator to periodically change the control in their extended model that allows the regulator to undertake lumpy infrastructure investment.

look to develop at any future date. Suppose that we denote $x(0)$ as x, and T as the time of development here and subsequently. The problem for an individual landowner is to determine the point at which it is optimal to develop, so as to maximize the expected net present value of one unit of vacant land, which is given by

$$E_0\left[\int_0^T e^{-\rho\tau}\gamma d\tau + \int_T^\infty e^{-\rho\tau}(\eta + x(\tau) - \theta D - nk)d\tau - e^{-\rho T}F\right]. \tag{6}$$

Equation (6) indicates that the expected net present value of returns to the unit plot of land at a particular location D is the sum of the returns to vacant land received until time T, plus the expected present value of land rent beginning at the time of development, less the expected present value of the sum of the cost paid for the public good and the up front developing costs.

We can rewrite Equation (6) as:

$$\frac{\gamma}{\rho} + E_0\left[\int_T^\infty e^{-\rho\tau}(\eta + x(\tau) - \theta D - nk - \gamma)d\tau - e^{-\rho T}F\right]. \tag{7}$$

In Equation (7), the first term is the expected present value of agricultural rents, assuming that the vacant land is never converted to urban use, while the remaining term is the developer's option value to delay development. The problem for the developer is then equivalent to choosing an optimal date to maximize this option value and, therefore, can be written as:

$$V(x) = \max_{T} E_0 \left[\int_T^{\infty} e^{-\rho\tau} [\eta + x(\tau) - \theta D - nk - \gamma] d\tau - e^{-\rho T} F \right]. \tag{8}$$

The solution for $V(x)$ must satisfy the fundamental differential equation of optimal stopping given by:

$$\frac{1}{2}\sigma^2 \frac{\partial^2 V(x)}{\partial x^2} + \alpha \frac{\partial V(x)}{\partial x} - \rho V(x) = 0. \tag{9}$$

Substituting the term $e^{\beta x}$ into Equation (9) yields the quadratic equation given by:

$$\phi(\beta) = -\frac{1}{2}\sigma^2\beta^2 - \beta\alpha + \rho = 0. \tag{10}$$

The solution to Equation (9) is then given by:

$$V(x) = A_1 e^{\beta_1 x} + A_2 e^{\beta_2 x}, \tag{11}$$

where A_1 and A_2 are constants to be determined, and β_1 and β_2 are, respectively, the larger and smaller roots of Equation (10):

$$\begin{aligned} \beta_1 &= -\frac{\alpha}{\sigma^2} + \frac{1}{\sigma^2}\sqrt{\alpha^2 + 2\rho\sigma^2} \quad > 0, \\ \beta_2 &= -\frac{\alpha}{\sigma^2} - \frac{1}{\sigma^2}\sqrt{\alpha^2 + 2\rho\sigma^2} \quad < 0. \end{aligned} \tag{12}$$

A landowner chooses the timing of development, which is characterized by x^*, the critical value of x that triggers property development. This critical value, and A_1 and A_2 in Equation (11), are obtained from the boundary conditions given by:

$$\lim_{x \to -\infty} V(x) = 0, \tag{13}$$

$$V(x^*) = \frac{\eta}{\rho} + \frac{x^*}{\rho} + \frac{\alpha}{\rho^2} - \frac{\theta D}{\rho} - \frac{\gamma}{\rho} - \frac{nk}{\rho} - F, \tag{14}$$

and

$$\frac{\partial V(x^*)}{\partial x} = \frac{1}{\rho}. \tag{15}$$

Equation (13) is the limit condition, which states that the value of the option to develop vacant land is worthless, as the state of nature is extremely poor, i.e., when the stochastic factor x approaches minus infinity. Equation (14) is the value-matching condition, which states that, at the optimal timing of development, a landowner should be indifferent as to whether vacant land is developed or not. Equation (15) is the smooth-pasting condition, which requires that the landowner not obtain any arbitrage profits from deviating from the optimal timing of development.

Solving Equations (13) to (15) simultaneously yields:

$$A_1 = \frac{1}{\rho\beta_1} e^{-\beta_1 x^*}, \tag{16}$$

$$A_2 = 0, \tag{17}$$

and

$$x^*(D,n) = \gamma + \rho F + kn + \theta D - \eta - \frac{\alpha}{\rho} + \frac{1}{\beta_1}. \tag{18}$$

Differentiating $x^*(D,n)$ with respect to its underlying parameters yields the following results.

Proposition 1. Developers will delay development ($x^*(D,n)$ increases) if: (*i*) they incur larger development costs (F increases); (*ii*) they receive more rents from vacant land (γ increases); (*iii*) they expect urban rents to become more volatile (σ increases); (*iv*) the regulator increases the stock of infrastructure (k increases); (*v*) urban residents incur larger commuting costs (θ increases); (*vi*) they expect urban rents to increase less rapidly over time (α decreases); (*vii*) their land is located far away from the CBD (D increases); and (*viii*) city population grows (n increases).

The reason for Proposition 1 is as follows. When deciding whether to develop, developers must compare the opportunity cost from delaying development (including the option value from waiting and the value of vacant land), with the net value from development. They will delay development either because the opportunity cost increases (scenarios (ii) and (iii)), the net value from development decreases (scenarios (i), (iv), (v), (vii), and (viii)), or the opportunity cost is decreased by less than the decrease in the net value from development (scenario (vi)).

Substituting A_1 given by Equation (16) and A_2 given by Equation (17) into Equation (10) yields the option value from waiting as given by:

$$V(x) = \frac{1}{\rho\beta_1} e^{\beta_1 (x - x^*(D,n))}. \tag{19}$$

The value of vacant land and that of urban land are thus respectively given by

$$V_a(x, D, n) = \frac{\gamma}{\rho} + \frac{1}{\rho\beta_1} e^{\beta_1 (x - x^*(D,n))}, \text{ if } \; x < x^*(D,n), \tag{20}$$

and

$$V_u(x, D, n) = \frac{\eta}{\rho} + \frac{x}{\rho} + \frac{\alpha}{\rho^2} - \frac{\theta D}{\rho} - \frac{nk}{\rho} - F, \text{ if } \; x \geq x^*(D,n).$$

Having determined the optimal development strategy (x^*) at a given location D, we now follow Fujita (1982) and Turnbull (1988) and ask: Given a state of nature denoted by $x(t)$, what will be the location $D^*(x(t))$, which will be associated with the state-contingent optimal timing of development? A developer, however, understands the urban growth process and acts with rational expectations. In particular, he knows that the city grows outward over time and thus realizes that his land will lie at the city's boundary at the time of conversion. Since the city is linear, the city's population n is thus equal to the distance to the boundary, given that each resident consumes one unit of land. In other words, in equilibrium when land at location D is converted, the city's population n will be equal to D. We can thus impose $n = D$, replace x^* with $x(t)$ and D with $D_d^*(x(t))$, and then transform Equation (18) into:

$$D_d^*(x(t)) = \frac{1}{(\theta + k)}[x(t) - a_0], \tag{21}$$

where $a_0 = \rho F + \gamma + \frac{1}{\beta_1} - \frac{\alpha}{\rho} - \eta$, and it is required that $x(t) > a_0$.

We may interpret Equation (21) as follows. Suppose that the state of nature $x(t)$

and the distance from the CBD, D, is such that $x(t)$ is greater than the x^* given by Equation (18). Some landowners will then develop vacant land further away from the CBD, increasing D until x^* rises to $x(t)$. Equivalently, if the distance from the CBD, D, is less than the $D_d^*(x(t))$ given by Equation (21), then some landowners will develop vacant land further away from the CBD until D is increased to the point $D_d^*(x(t))$.[6] Since the city grows outward, $D_d^*(x(t))$ is, in fact, the equilibrium city boundary at time t.

Differentiating $D_d^*(x(t))$ with respect to its underlying parameters yields the following results.

Proposition 2. Without any regulation, the city boundary will be contracted ($D_d^*(x(t))$ decreases) if: (*i*) developers incur larger development costs (F increases); (*ii*) landowners receive higher agricultural rents (γ increases); (*iii*) developers expect urban rents to be more volatile (σ increases); (*iv*) the regulator increases the stock of infrastructure (k increases); (*v*) urban residents incur larger commuting costs (θ increases); (*vi*) developers expect urban rents to increase less rapidly over time (α decreases).

The results of Proposition 2 come directly from Proposition 1. Specifically, if

[6] The above argument resembles that of Pindyck (1988), when he explains the timing decision for undertaking a continuous investment project. See also Lee and Jou (2010) for a thorough discussion of this issue.

developers delay development, then the equilibrium city boundary will also shrink. For example, suppose that developers expect urban rents to become more volatile. As indicated by Proposition 1(iii), developers will then delay development, because their option value from waiting becomes more valuable. As a result, the equilibrium city boundary will shrink, as developers are less inclined to transform vacant land for urban use.

We can compare the above case with the first-best case in which the public good is priced at the marginal cost, $2nk$. Following similar procedures as above yields the first-best growth boundary as given by

$$D_f^*(x(t)) = \frac{1}{(\theta + 2k)}[x(t) + \eta + \frac{\alpha}{\rho} - \frac{1}{\beta_1} - \gamma - \rho F]. \tag{22}$$

It is obvious that the equilibrium growth boundary shown in Equation (21) is broader than the first-best level, as shown in Equation (22). Consequently, it is possible for the regulator to implement a growth boundary control that enhances the social welfare.

Since consumer utility is exogenous in our model, the social planner's goal is then to choose an efficient control to maximize the total land value, net of the implementation cost. To derive total land value, the value expression in Equation (6) is integrated across all locations D in the planner's jurisdiction. The city boundary set by the social planner is denoted by D_c, and thus city population, n will be equal to D_c, given that the open city is linear. Imposing this constraint yields the objective function of the social planner as given by:

$$\underset{D_c}{Max}\, W(D_c) = \underset{Dc}{Max}\, [\int_0^{D_c} V_u(x, D, D_c) dD \tag{23}$$

$$+\int_{D_c}^{B} V_a(x, D, D_c) dD - \varepsilon(D_d - D_c)^2], \quad \varepsilon > 0,$$

where the outer boundary of the planner's jurisdictions, denoted by B, is assumed to be sufficiently large such that $B > D_c$. Inside the brackets of Equation (23), the first integral is total urban land value, the second integral is total agricultural land value, and the last term is the implementation cost, which indicates that the cost for the regulator to monitor the vacant landowners is an increasing convex function of the distance between the equilibrium city boundary and the controlled boundary. This is plausible because the farther the distance of a parcel of land located from the controlled city boundary, the more the costs (e.g., transportation costs) for the regulator to monitor the owner of this parcel of land.

The efficient growth control, denoted by D_c^*, is found by totally differentiating $W(D_c)$ in Equation (23) with respect to D_c and setting the result equal to zero. This yields:

$$\frac{dW}{dD_c} = V_u(x, D_c, D_c) - V_a(x, D_c, D_c) - \frac{k}{\rho} D_c \tag{24}$$

$$-\frac{k}{\rho}\int_{D_c}^{B} e^{\beta_1(x-x^*(D,D_c))}dD + 2\varepsilon(D_d - D_c) = 0\,.$$

Differentiating dW / dD_c in Equation (24) with respect to D_c yields

$$\Delta = \frac{d^2W}{dD_c{}^2} = \frac{(\theta+2k)}{\rho}[-1+e^{\beta_1(x-x^*(D_c,D_c))}] \tag{25}$$

$$+\frac{k^2}{\rho\theta}[e^{\beta_1(x-x^*(D_c,D_c))} - e^{\beta_1(x-x^*(B,D_c))}] - 2\varepsilon\,,$$

We must restrict ε to be positive in order to derive an interior solution for D_c. The reason is as follows. Without any implementation cost ($\varepsilon = 0$), Equation (25) indicates that $\Delta > 0$ because $x = x^*(D_d^*, D_d^*) > x^*(D_c^*, D_c^*)$ (since D_d^* is assumed to be larger than D_c^*), such that the first two terms on the right-hand side of Equation (25) are both positive. In other words, without any implementation cost, the objective function for the social planner shown in Equation (23) becomes a strictly convex function of the level of the boundary control, D_c. As a result, the regulator will either set the efficient boundary at $D_c^* = 0$ or $D_c^* = D_d^*$, and thus the problem in consideration becomes a trivial one. We thus consider only the case where $\varepsilon > 0$.

For this case, Equation (24) indicates that if a regulator slackens the boundary control (D_c increases), then the society as a whole receives the value of urban land at location D_c, $V_u(x, D_c, D_c)$, but suffers the cost of the sum of the following three items: $V_a(x, D_c, D_c)$, $\frac{kD_c}{\rho}$, $\frac{k}{\rho}\int_{D_c}^{B} e^{\beta_1(x-x^*(D,D_c))}dD$, and $-2\varepsilon(D_d - D_c)$. The first term represents the loss of the value of agricultural land at location D_c that is transformed to urban use, the second term represents the costs borne by all existing urban residents as new urban residents flow in. The third term represents the loss of the option value borne by all owners of vacant land because as the regulator slackens the boundary control, these landowners will respond to delay development (since $\partial x^*(D, D_c) / \partial D_c > 0$), thus reducing the option value from waiting.[7] Finally, the last term represents the cost savings associated with slackening the control policy.

We may compare D_c^* with D_d^* in Equation (21), the equilibrium city boundary, which is determined by the condition that at the city boundary, urban land value is just equal to vacant land value, i.e., $V_u(x, D_d^*, D_d^*) = V_a(x, D_d^*, D_d^*)$. Equation (24) suggests that the efficient city boundary is set by the condition $V_u(x, D_c, D_c) = V_a(x, D_c, D_c) + \dfrac{kD_c}{\rho} + \dfrac{k}{\rho}\int_{D_c}^{B} e^{\beta_1(x - x^*(D, D_c))} dD - 2\varepsilon(D_d - D_c)$, which implies that if the regulator sets the city boundary at the equilibrium level, then the marginal cost of the control will be larger than the marginal benefit (provided that ε is sufficiently small). Consequently, the regulator can set an efficient boundary at a lower level to enhance the social welfare.

Differentiating Equation (24) with respect to its underlying parameters yields the

[7] This effect resembles those of the other forms of land use regulation policies, such as density ceiling control (Jou and Lee, 2007; Turnbull, 1991), a threat to expropriate development rights (Riddiough, 1997; Turnbull, 2002), and the threat of a development moratorium (Turnbull, 2004b). See a thorough review by Turnbull (2005).

following results.

Proposition 3. The regulator should tighten the boundary control if (i) developers incur larger costs to develop vacant land; (ii) landowners receive higher agricultural rents; (iii) developers expect urban rents to evolve more volatile; and (iv) it is less costly for the regulator to implement the control policy.

Proof: See Appendix A.

The reason for Proposition 3 is as follows. When the development cost increases, a regulator should tighten the boundary control so as to avoid some landowners to incur more costs to convert agricultural land into urban use. When agricultural rents increase, the regulator should also tighten the boundary control because the opportunity cost for landowners to transform vacant land into urban use increases. As urban rents become volatile, those owners of vacant land will delay development such that the regulator can tighten the boundary control to accommodate this delaying effect. Finally, as the implementation cost decreases, the regulator naturally has more incentives to tighten the control.

We may compare our results with those of Ding et al. (1999), who assume that there exists no uncertainty, i.e., $\sigma = 0$. Substituting this condition into Equation (10) yields $\beta_1 = \rho / \alpha$, and thus the critical level of $x(t)$ that triggers development, as shown in Equation (18), and the equilibrium city boundary, as shown in Equation (21), will respectively become

$$x^*(D,n) = \gamma + \rho F + nk + \theta D - \eta \,, \tag{26}$$

and

$$D_d^*(x(t)) = \frac{1}{(\theta + k)}[x(t) + \eta - \gamma - \rho F]\,. \tag{27}$$

We provide a rigorous proof for $x^*(D)$ shown in Equation (27) in Appendix B.

Figure 1 illustrates the main difference between our result and that of Ding et al. (1999) for the case in the absence of any uncertainty. In that Figure, the horizontal axis, $x(t)$, has one-to-one correspondence with calendar time t because $x(t) = x(0) + \alpha t$. The path D_d^* is that shown in Equation (27), and D_c^* is the D_c that satisfies Equation (24). Ding et al. design the efficient city boundary as the one that imposes no control before a critical date, t^*, and then sets the city boundary at $D_d^*(x(t^*))$ afterwards. Without any uncertainty, both of ours and Ding et al.'s are two different ways to control the city boundary. Ours is similar to that proposed by Brueckner (1990), which is more lenient to those landowners whose land is located far away from the CBD. Besides, the control also accommodates demand for urban land because we assume that demand for urban land increases over time such that urban rents also increase over time.

When there is uncertainty, it is not plausible for a regulator to directly control the development timing such that it is infeasible for the regulator to implement the control set in Ding et al. It is, however, still possible to implement the control as addressed in our framework, as shown in the next section.

IV. Numerical Analysis

We employ plausible parameter values to investigate how the characteristics of demand and supply conditions in the real estate market affect the design of efficient boundary controls. The benchmark values we choose are as follows. The fixed component of urban rents is equal to 2.0 ($\eta = 2.0$), the return to vacant land is fixed at 0.1 unit ($\gamma = 0.1$), and the development cost is set at 1 unit ($F = 1$). The factor that shifts urban rents is expected to grow at 3% per year ($\alpha = 3\%$), and the volatility of this shift factor is equal to 20% per year ($\sigma = 20\%$). As the distance from the CBD increases by one mile, urban rents decrease by one-tenth unit ($\theta = 0.1$). We also assume that the stochastic factor that affects urban rents, i.e., $x(t)$, is equal to 1. All developer are risk-neutral and face a discount rate equal to 10% per year ($\rho = 10\%$), and the stock of infrastructure is normalized at one unit ($k = 1$). The outer boundary of the planner's jurisdictions is set at 15 miles ($B = 15$). Finally, the parameter associated with the implementation cost, ε, is set at 1,000 ($\varepsilon = 1,000$).

Given these parameter values, Table 1 shows that the equilibrium city boundary is equal to 2.253 miles, i.e., $D_d^* = 2.253$; the efficient city boundary is equal to 2.225 miles, i.e., $D_c^* = 2.225$, and the total social welfare W^* is equal to 91.37, which is 0.85% greater than its counterpart in the absence of any regulation, i.e., 90.60. Table 1 also shows that F changes in the region (0.5, 1.5), γ changes in the region (0, 0.2), σ changes in the region (15%, 25%), k changes in the region (0.5, 1.5), θ changes in the region (0.05, 0.15), α changes in the region (1%, 3%), and ε changes in the region (500, 1500), holding all other parameters at their benchmark values.

Several important findings derived from Table 1. First, for all exogenous forces except for the implementation cost factor (ε), increases in these factors have the same qualitative effects on both the equilibrium city boundary and the efficient city boundary. Specifically, similar to Ding et al. (1999), we find that an increase in either the development cost, the agricultural rent, or the stock of infrastructure decreases the optimal size of the city.[8] Furthermore, we also find that a decrease in the commuting cost expands the optimal city size.

[8] The last scenario will hold in the framework of Ding et al. if they assume that the marginal cost of providing the public good is increasing (rather than decreasing) in the stock of infrastructure.

We contribute to the literature by furthermore providing the following testable implications: The regulator will set a more stringent boundary control policy if developers expect urban rents to grow less rapidly or to be volatile, or if the regulator spends less to implement the control policy.

To explain our policy implications more clearly, we use the same benchmark parameter values to generate Figure 2 (expect for $x(t)$), which presents a series of stochastic rents $x(t)$, the corresponding equilibrium city boundary, D_d^*, and the efficient city boundary, D_c^*. Starting from $t = 0$ and $x(0) = 1$, the figure shows that the corresponding D_d^* and D_c^* are equal to 1.344 and 1.315, respectively. Both the equilibrium city boundary D_d^* and the efficient city boundary D_c^* will not increase unless $x(t)$ exceeds all its historical values. This occurs at those years: $t =$ 0.1, 0.4, 1.3, 1.5, 1.9, 2.3, 3.1, 8.4, 9.3, 9.4, and 9.5. In other words, the regulator will not alter the city boundary control when urban rents move downward, but will slacken the control when urban rents move upward. This type of control, which pushes the city boundary outward as time goes by,[9] is also consistent with the existing land-use planning system that requires local governments to periodically change city boundaries as population grows (See, e.g., Ding et al. (1999) and Jaeger and Plantinga (2007)).

[9] Cumming (2010) reports how property speculators buy up land on the Auckland city outskirts and waiting until the time was ripe to develop. City governments are thus usually forced to broaden the boundary to meet the increase in demand for urban land.

V. Conclusion

This article extends Ding et al. (1999) to investigate the design of growth boundary controls in an open city in a framework where a congestible public good is priced at average cost. Unlike Ding et al., we consider that urban rents evolve stochastically over time and that the regulator can slacken the boundary control when urban rents move upward. We provide three novel testable implications, namely, the regulator will set a more stringent boundary control policy if (i) developers expect urban rents to grow less rapidly; (ii) developers expect urban rents to be more volatile; and (iii) it costs less for the regulator to implement the control policy.

Our article considers an open city model and, thus, suffers the shortcomings of this consideration. As Brueckner (1990) indicates, this implies that population pressure and the resulting excess demand for housing plays no role in determining the market impact of growth controls. We can follow the supply-restriction model, as outlined in both Brueckner (1995) and Helsley and Strange (1995), to take this into account. Furthermore, we may follow Brueckner and Lai (1996) to distinguish renters from landlords in the model. We may then compare the results of this article with those of the more generalized model.

Appendix A:

Totally differentiating Equation (24) with respect to F , γ , σ , and ε rearranging yields

$$\frac{\partial D_c^*}{\partial F} = \frac{\Delta_1}{-\Delta} < 0\,, \tag{A1}$$

$$\frac{\partial D_c^*}{\partial \gamma} = \frac{\Delta_2}{-\Delta} < 0\,, \tag{A2}$$

$$\frac{\partial D_c^*}{\partial \sigma} = \frac{\Delta_3}{-\Delta} < 0\,, \tag{A3}$$

and

$$\frac{\partial D_c^*}{\partial \varepsilon} = \frac{\Delta_4}{-\Delta} > 0\,, \tag{A4}$$

where as shown in Equation (25), $\Delta < 0$, and

$$\Delta_1 = \partial(dW / dD_c) / \partial F = \beta_1 (x - x^*(D_c, D_c) - \rho k D_c) \frac{-2\varepsilon\rho}{(\theta + k)} < 0\,,$$

$$\Delta_2 = \partial(dW / dD_c) / \partial \gamma = -\frac{\beta_1}{\rho} (x - x^*(D_c, D_c) - \rho k D_c) \frac{-2\varepsilon}{(\theta + k)} < 0\,,$$

$$\Delta_3 = \frac{1}{\beta_1} \frac{\partial \beta_1}{\partial \sigma} [(\frac{1}{\rho} + \frac{k}{\theta})(x^*(D_c, D_c) - x) e^{-\beta_1 (x^*(D_c, D_c) - x)}$$

$$- \frac{k}{\theta} (x^*(B, D_c) - x) e^{-\beta_1 (x^*(B, D_c) - x)}] + \frac{1}{(\theta + k)\beta_1^2} < 0\,,$$

given that $\partial\beta_1 / \partial\sigma < 0$,

and

$$\Delta_4 = 2(D_d - D_c) > 0\,.$$

Q.E.D.

Appendix B:

In the absence of any uncertainty $x(\tau) = x + \alpha\tau$. Substituting this into Equation (8) yields the objective function of the developer as given by

$$V(x) = \max_{T} U(T, x) = [\frac{1}{\rho}(\eta + x(\alpha T + \frac{\alpha}{\rho} - \theta D - \gamma - nk)) - F]e^{-\rho T}. \qquad \text{(B1)}$$

Intuitively, a landowner needs to decide to either develop immediately in the initial period or delay development. The former occurs if $\alpha \le 0$, while the latter occurs if $\alpha > 0$. Suppose that the landowner develops at a time $T = T^* > 0$. Differentiating Equation (A1) with respect to T, and then setting the result equal to zero yields

$$\frac{\partial U(T^*, x)}{\partial T} = [\rho F + nk - (\eta + x - \theta D - \gamma) - \alpha T^*]e^{-\rho T^*} = 0. \qquad \text{(B2)}$$

The second-order condition is given by

$$\frac{\partial^2 U(T^*, x)}{\partial T^2} = -\alpha e^{-\rho T^*} < 0, \qquad \text{(B3)}$$

which holds if $\alpha > 0$.

Equation (B2) indicates that the interior solution for T is given by

$$T^* = \frac{1}{\alpha}[\gamma + \rho F + nk + \theta D - \eta - x] > 0, \qquad \text{(B4)}$$

which requires that

$$\gamma + \rho(F + nk) + \theta D > \eta + x. \qquad \text{(B5)}$$

Imposing $T^* = 0$ and replacing x by x^* yields x^* as shown in Equation (26).

Substituting $T = T^*$ into Equation (B1) yields the option value of waiting as given by

$$V(x) = \frac{\alpha}{\rho^2} e^{-\rho T^*} . \tag{B6}$$

Q.E.D.

Table 1: Efficient Boundary Controls and Efficient Impact Fees.

Benchmark Case: $F=1$, $\theta=0.1$, $\sigma=0.2$, $k=1$, $\gamma=0.1$, $\alpha=0.03$, $\eta=2$, $x(t)=1$, $\varepsilon=1{,}000$, $B=15$, $\rho=0.1$.

	Variation in F				
	0.50	0.75	1.00	1.25	1.50
D_c^*	2.270	2.247	2.225	2.201	2.179
D_d^*	2.298	2.276	2.253	2.230	2.208
W^*	92.76	92.06	91.37	90.69	90.01
D_c^* / D_d^*	0.988	0.988	0.987	0.987	0.987
	Variation in γ				
	0.00	0.05	0.10	0.15	0.20
D_c^*	2.315	2.270	2.225	2.179	2.134
D_d^*	2.344	2.298	2.253	2.208	2.162
W^*	79.16	85.26	91.37	97.51	103.68
D_c^* / D_d^*	0.988	0.988	0.987	0.987	0.987
	Variation in σ				
	0.150	0.175	0.200	0.225	0.250
D_c^*	2.323	2.274	2.225	2.174	2.124
D_d^*	2.348	2.301	2.253	2.205	2.156
W^*	82.51	86.80	91.37	96.19	101.22
D_c^* / D_d^*	0.989	0.988	0.987	0.986	0.985
	Variation in k				
	0.50	0.75	1.00	1.25	1.50
D_c^*	4.117	2.895	2.225	1.799	1.502
D_d^*	4.131	2.916	2.253	1.836	1.549
W^*	123.95	102.78	91.37	84.43	79.95
D_c^* / D_d^*	0.997	0.993	0.987	0.980	0.970
	Variation in θ				
	0.050	0.075	0.100	0.125	0.150
D_c^*	2.318	2.271	2.225	2.179	2.134
D_d^*	2.360	2.305	2.253	2.203	2.155
W^*	109.95	99.41	91.37	85.16	80.11
D_c^* / D_d^*	0.982	0.985	0.987	0.989	0.990

	Variation in α				
	0.01	0.02	0.03	0.04	0.05
D_c^*	2.157	2.193	2.225	2.251	2.275
D_d^*	2.182	2.220	2.253	2.282	2.307
W^*	75.73	83.16	91.37	100.33	109.98
D_c^* / D_d^*	0.989	0.988	0.987	0.987	0.986
	Variation in ε				
	500	750	1000	1250	1500
D_c^*	2.193	2.214	2.224	2.230	2.234
D_d^*	2.253	2.253	2.253	2.253	2.253
W^*	92.23	91.65	91.37	91.21	91.11
D_c^* / D_d^*	0.973	0.983	0.987	0.990	0.992

Note: The terms, F, γ, σ, k, θ, α, η, ε, B, ρ, D_c^*, D_d^*, and W^* denote the development costs, the return to vacant land, the volatility of the stochastic urban rent, the stock of infrastructure, the commuting costs per unit distance to the CBD, the expected growth rate of the stochastic rent, the fixed component of urban rents, the parameter associated with the implementation cost, the outer boundary of the planner's jurisdiction, the discount rate per year, the efficient city boundary, the equilibrium city boundary, and total social welfare.

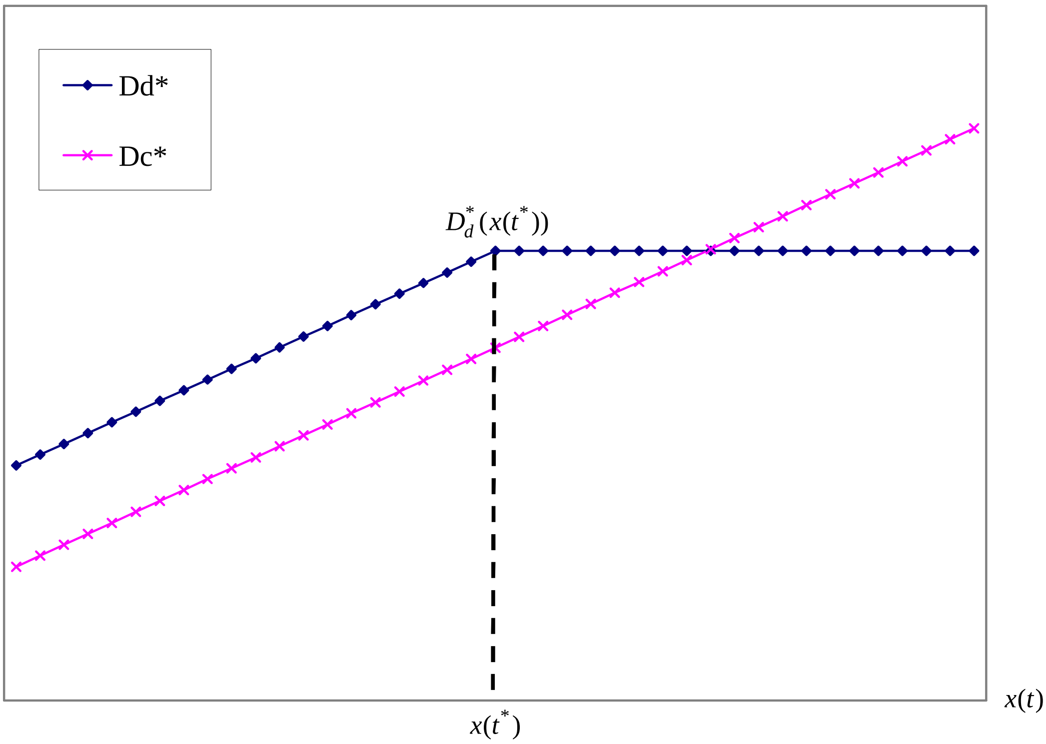

Figure 1: The efficient city boundary control in the absence of any uncertainty. This graphs shows that the city boundary designed by Ding et al. (1999) is to impose no control before $t = t^*$ such that the efficient city boundary is the same as the equilibrium city boundary, $D_d^*(x(t))$. Immediately after $t = t^*$, the regulator then imposes a fixed level of city boundary at $D_d^*(x(t^*))$. By contrast, the efficient city boundary in our framework, $D_c^*(x(t))$, is always lower than the equilibrium level, $D_d^*(x(t))$.

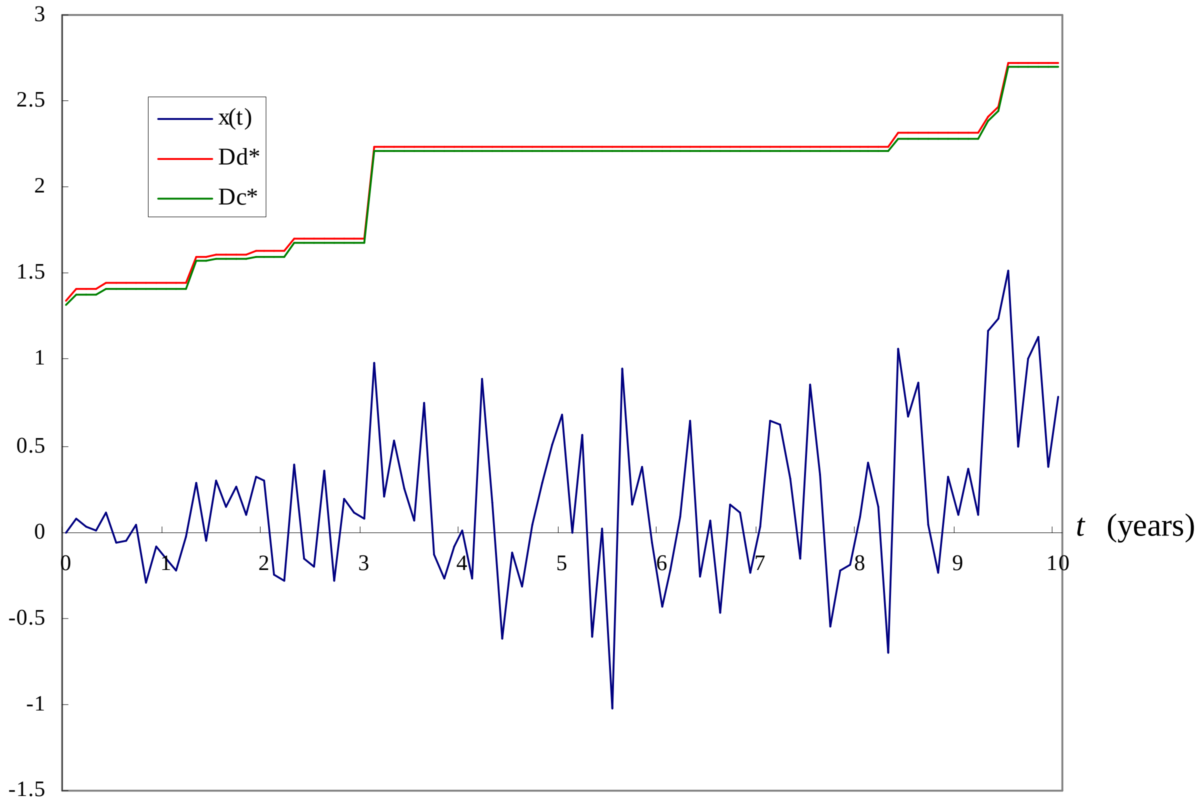

Figure 2: The Equilibrium City Boundary, D_d^*, the Efficient City Boundary, D_c^*, and the Stochastic Component of Urban Rents, $x(t)$, as a Function of Calendar Year, t. Both D_d^* and D_c^* will not change unless $x(t)$ exceeds $x(\tau)$, for all $\tau < t$.

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